High Energy Facilities
Advanced Projects

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York 11973

RHIC-3

RHIC Technical Note No. 3

THE DOUBLE-TUNED COUPLING CIRCUIT

THEORY AND DESIGN CRITERIA

High Energy Facilities
Advanced Projects

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York 11973

RHIC Technical Note No. 3

THE DOUBLE-TUNED COUPLING CIRCUIT THEORY AND DESIGN CRITERIA

M. Puglisi

August 20, 1984

INTRODUCTION

The stability of an amplifier may depend upon the output impedance of its driving circuit.

A very simple procedure is presented here for designing the simplest interstage circuit based upon the inductive coupling

1. THE IMPEDANCE TRANSFORMATION

The simplest equivalent scheme of the coupling between the driver and the final amplifier is given in Fig. 1.

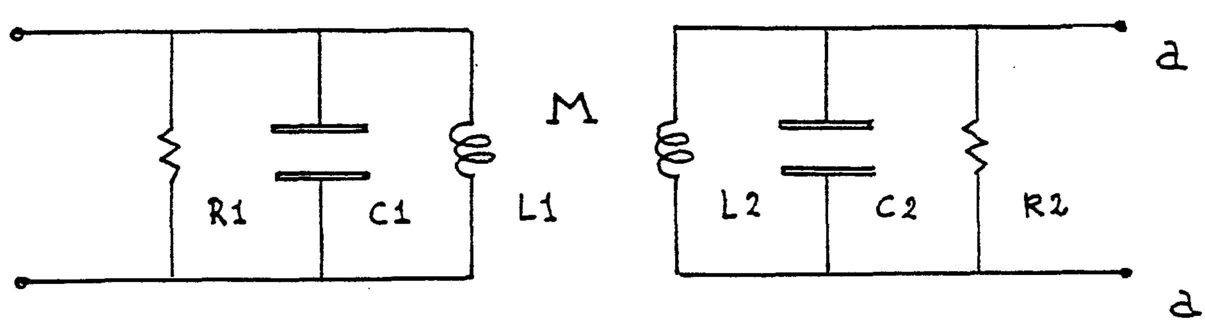

Fig. 1. Equivalent scheme of the coupling circuit. The output impedance of the driver and the input impedance of the driven circuit are explicitly taken into account.

where R_1, R_2, C_1, C_2 are the four parameters that define the boundary conditions of the problem* while L_1, L_2 and M are the parameters to be determined.

Let us consider the circuit drawn in Fig. 2 that is the same circuit given in Fig. 1 without the elements depending upon the final tube.

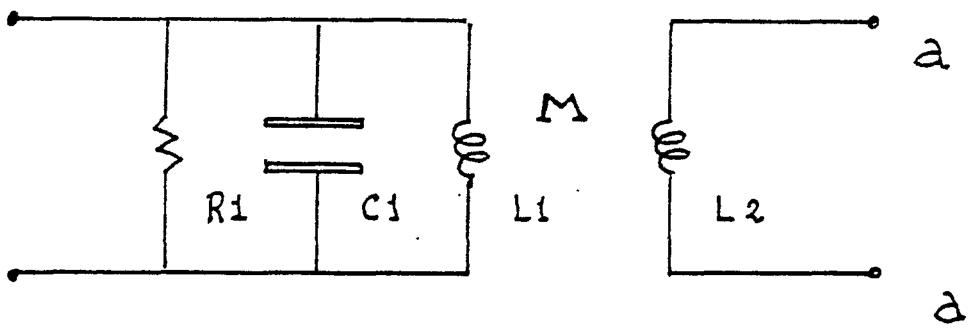

Fig. 2. Equivalent scheme of the coupling circuit as is seen by the amplifier.

After some tedious algebra the admittance Y appearing between the terminals aa can be written as follows:

$$Y = \frac{R_1(1-\omega^2 L_1 C_1) + j\omega L_1}{-\omega^2 L_1 L_2 (1-K^2) + j\omega L_2 R_1 \left[1-\omega^2 L_1 C_1 (1-K^2)\right]} \tag{1}$$

where, as usual, K is the coupling coefficient.

Before going further we consider two special cases:

a) $K = 0$.

In this case it turns out that:

$$Y(0) = \frac{1}{j\omega L_2}$$

*It should be noted that R_1, C_1, and R_2, C_2 must be considered as equivalent circuit elements because they depend upon both the characteristics of the tubes involved and the values of the added elements.

which is consistent with the very nature of this coupling. When K=0 there is no coupling and only the inductor L_2 appears between the terminals aa.

b) K = 1.

In this extreme case we obtain:

$$Y(1) = \frac{L_1}{L_2}\left(\frac{1}{R_1} - j\,\frac{1-\omega^2 L_1 C_1}{\omega L_1}\right) \tag{2}$$

which is again consistent with the physics of the coupling because if K=1 then the primary circuit impedance appears across the output terminal throughout an ideal transformer whose transforming ratio is equal to $\sqrt{L_1/L_2}$.

2. FIRST APPROXIMATION DESIGN

Because we are always interested in the high coupling case the previous limiting case (K=1) will be considered as a guideline.

Accordingly with the assumptions already made the final amplifier equivalent input admittance Y_F is as follows.

$$Y_F = \frac{1}{R_2} + j\omega C_2 \tag{3}$$

and is supposed to be connected across the aa terminals shown in Fig. 2.

Consequently the total input admittance Y_T seen by the final tube is given by Eq. (4).

$$Y_T = \frac{1}{R_1}\frac{L_1}{L_2} + \frac{1}{R_2} + j\left(\omega C_2 - \frac{1-\omega^2 L_1 C_1}{\omega L_2}\right) \tag{4}$$

If ω_o is the working radian frequency then the tuning condition is as follows:

$$L_1 C_1 + L_2 C_2 = \frac{1}{\omega_o^2} \tag{5}$$

and it is very important to recognize that the same condition holds for the primary circuit.

If we call Req the value of the resistor that should appear across the input of the final tube (being equal to zero the imaginary part of the admittance) then from Eq. (4) we should have.

$$\begin{cases} \frac{1}{R_1}\frac{L_1}{L_2} + \frac{1}{R_2} = \frac{1}{Req} \\ \\ L_1C_1 + L_2C_2 = \frac{1}{\omega_o^2} \end{cases} \tag{6}$$

Solving we obtain:

$$L_1 = \frac{\frac{R_1}{Req} - \frac{R_1}{R_2}}{\omega_o^2\left(C_2+\left(\frac{R_1}{Req} - \frac{R_1}{R_2}\right)C_1\right)} \tag{7}$$

$$L_2 = \frac{1}{\omega_o^2\left(C_2+\left(\frac{R_1}{Req} - \frac{R_1}{R_2}\right)C_1\right)} \tag{8}$$

It should be remembered that the values of L_1 and L_2 already calculated are true only for K=1. Moreover the resistors R_1, R_2, Req cannot be completely arbitrary as it is obvious both from the physics and from the formulae.

In the next paragraph it will be shown how to treat the case for K<1.

3. THE FINAL DESIGN

Now we take into account the fact that the coupling coefficient should be always less than 1.

Coming back to the general expression calculated for the impedance between the terminal aa we put:

$$Y = G + jB$$

where:

$$G = \frac{1}{R_1} \frac{K^2}{\frac{\omega^2 L_1 L_2}{R_1^2}(1-K^2)^2 + \frac{L_2}{L_1}\left(1-\omega^2 L_1 C_1 (1-K^2)\right)^2}$$

$$B = -\frac{1}{\omega L_1} \frac{(1-\omega^2 L_1 C_1)\left(1-\omega^2 L_1 C_1 (1-K^2)\right) + \frac{\omega^2 L_1^2}{R_1}\ (1-K^2)}{\frac{\omega^2 L_1 L_2}{R_1^2}(1-K^2)^2 + \frac{L_2}{L_1}\left(1-\omega^2 L_1 C_1 (1-K^2)\right)^2} \tag{9}$$

Following the procedure already seen we should write the boundary conditions for the whole circuit:

$$Y + Y_F = G + jB + \frac{1}{R_2} + j\omega C_2$$

this means that we have to solve for L_1 and L_2 the following system

$$\begin{cases} G + \frac{1}{R_2} = \frac{1}{Req} \\ \\ \omega C_2 + B = 0 \end{cases} \tag{10}$$

and it is easy to see that this might represent a formidable task.

Nevertheless if we are looking for a high value of K then the value of L_1 and L_2 calculated for K=1 (and the given values for C_1 and C_2) cannot be largely different from those which are appropriate for K<1.

Consequently if we will use the calculated values for L_1 and L_2 then we should adjust, by a very little amount, the values of C_1 and C_2 already assumed as boundary conditions.

This can be easily obtained from the system 10 as follows:

Calculate L_1 and L_2 with (7) and (8).

Define the quantity:

$$A = \frac{1}{K^2}\left(\frac{R_1}{Req} - \frac{R_1}{R_2}\right) \tag{11}$$

upon substituting into the system where now L_1 and L_2 are known quantities, we obtain:

$$\begin{cases} \dfrac{\omega^2 L_1 L_2}{R_1^2}(1-K^2)^2 + \dfrac{L_2}{L_1}\left(1-\omega^2 L_1 C_1 (1-K^2)\right)^2 = \dfrac{1}{A} \\ \\ \omega C_2 = \dfrac{A}{\omega L_1}\left\{(1-\omega^2 L_1 C_1)\left(1-\omega^2 L_1 C_1 (1-K^2)\right) + \dfrac{\omega^2 L_1^2}{R_1^2}(1-K^2)\right\} \end{cases} \tag{12}$$

This system can be easily solved for the new values of C_1 and C_2:

$$\begin{cases} C_1 = \dfrac{1}{\omega^2 L_1} * \dfrac{1 - \sqrt{K^2 - \left(\frac{\omega L_1}{R_1}(1-K^2)\right)^2}}{1-K^2} \\ \\ C_2 = \dfrac{1}{\omega^2 L_1} A \left\{(1-\omega^2 L_1 C_1)\left(1-\omega^2 L_1 C_1 (1-K^2)\right) + \dfrac{\omega^2 L_1^2}{R_1^2}(1-K^2)\right\} \end{cases}$$

4. CONCLUSION

We assume that the parameters R_1, R_2, Req and the coupling coefficient K are given as well as the working radian frequency ω_o.

Moreover, because the tubes that are to be used are also known, then we can make a reasonable guess for the values of the capacitors C_{1g} and C_{2g} that should appear in the circuit. (The letter g stays for guess).

Then we calculate the quantity A as already seen:

$$A = \frac{1}{K^2}\left(\frac{R_1}{Req} - \frac{R_1}{R_2}\right) > 0$$

From the guessed values C_{1g} and C_{2g} we obtain:

$$L_1 = \frac{1}{\omega_o{}^2 C_{1g}} \frac{AK^2}{\frac{C_{2g}}{C_{1g}} + AK^2} \tag{12}$$

$$L_2 = \frac{L_1}{AK^2} \tag{13}$$

Using the values of L_1 and L_2 we readjust C_{1g} obtaining C_1:

$$C_1 = \frac{1}{\omega_o{}^2 L_1} \frac{1 - \sqrt{K^2 - \left(\frac{\omega L_1}{R_1}(1-K^2)\right)^2}}{1-K^2} \tag{14}$$

once L_1, L_2, C_1 are known then:

$$C_2 = \frac{1}{\omega_o{}^2 L_1} A \left\{(1-\omega^2 L_1 C_2)\left(1-\omega^2 L_1 C_1 (1-K^2)\right) + \frac{\omega^2 L_1^2}{R_1{}^2} (1-K^2)\right)\} \tag{15}$$

and the problem is completely solved.

It should be considered that the procedure already shown is completely correct and that, if the actual coupling coefficient K is reasonably close to 1, then the values of C_1 and C_2 should come very close to the guessed values for C_{1g} and C_{2g}.

5. A NUMERICAL EXAMPLE

The above procedure was mainly developed for an interstage coupling of the RHIC machine. If the Eimac tube 3CX1500A7 is to be used then the prescribed values for the various parameter could be as follows:

ν_0 = 27 MHz;

R_1 = 2000 Ω

R_2 = 10,000 Ω

Req = 1,500 Ω

and as a reasonable guess we could assume

$$C_{1g} = C_{2g} = 60\text{ E-12 F.}$$

$$K = 0.9$$

from Eqs. (11), (12), (13) we obtain:

$$A = 1.399177,\ L_1 = 3.076\text{ E-7 H},\ L_2 = 2.714\text{ E-7 H.}$$

upon substituting those values into Eqs. (14) and (15) we obtain:

$$C_1 = 59.45\text{ E-12 F},\ C_2 = 67.37\text{ E-12 F.}$$

The ECAP program for checking the circuit behavior is given in Table 1 and the diagrams plotted on Figures 3 and 4 show the validity of the outlined procedure.

ACKNOWLEDGMENT

I would like to thank the staff of the Word Processing Center for an excellent job.

Table 1.

```
CAR.  AC ANALYSIS

 1.          AC ANALYSIS
 2.   B1     N(1,0),R=2000..I=1.
 3.   B2     N(1,0),C=59.45E-12
 4.   B3     N(1,0),L=3.0764E-7
 5.   B4     N(2,0),L=2.7146E-7
 6.   B5     N(2,0),C=67.37E-12
 7.   B6     N(2,0),R=10E3
 8.   M1     B(3,4),L=2.6009E-7
 9.          FREQUENCY=27E6
10.          EXECUTE

NO FATAL INPUT ERRORS DETECTED.  EXECUTI

11.          MODIFY
12.          FREQUENCY=26E6(+200)28E6
13.          PRINT,NV(1)
14.          PLOT,NV(1)
15.          PLOT,NV(2)
16.          EXECUTE

NO FATAL INPUT ERRORS DETECTED.  EXECUTI
```

```
ECAR.  AC ANALYSIS

 1.          AC ANALYSIS
 2.   B1     N(1,0),R=2000.
 3.   B2     N(1,0),C=59.45E-12
 4.   B3     N(1,0),L=3.0764E-7
 5.   B4     N(2,0),L=2.7146E-7
 6.   B5     N(2,0),C=67.37E-12
 7.   B6     N(2,0),R=10E3.I=1.
 8.   M1     B(3,4),L=2.6009E-7
 9.          FREQUENCY=27E6
10.          EXECUTE

NO FATAL INPUT ERRORS DETECTED.  EXE

11.          MODIFY
12.          FREQUENCY=26E6(+200)28E6
13.          PRINT,NV(1)
14.          PLOT,NV(1)
15.          PLOT,NV(2)
16.          EXECUTE

NO FATAL INPUT ERRORS DETECTED.  EXE
```

Listing of the ECAP programs for calculating the input impedance seen by the driver (right) and the input impedance seen by the driven circuit (left).

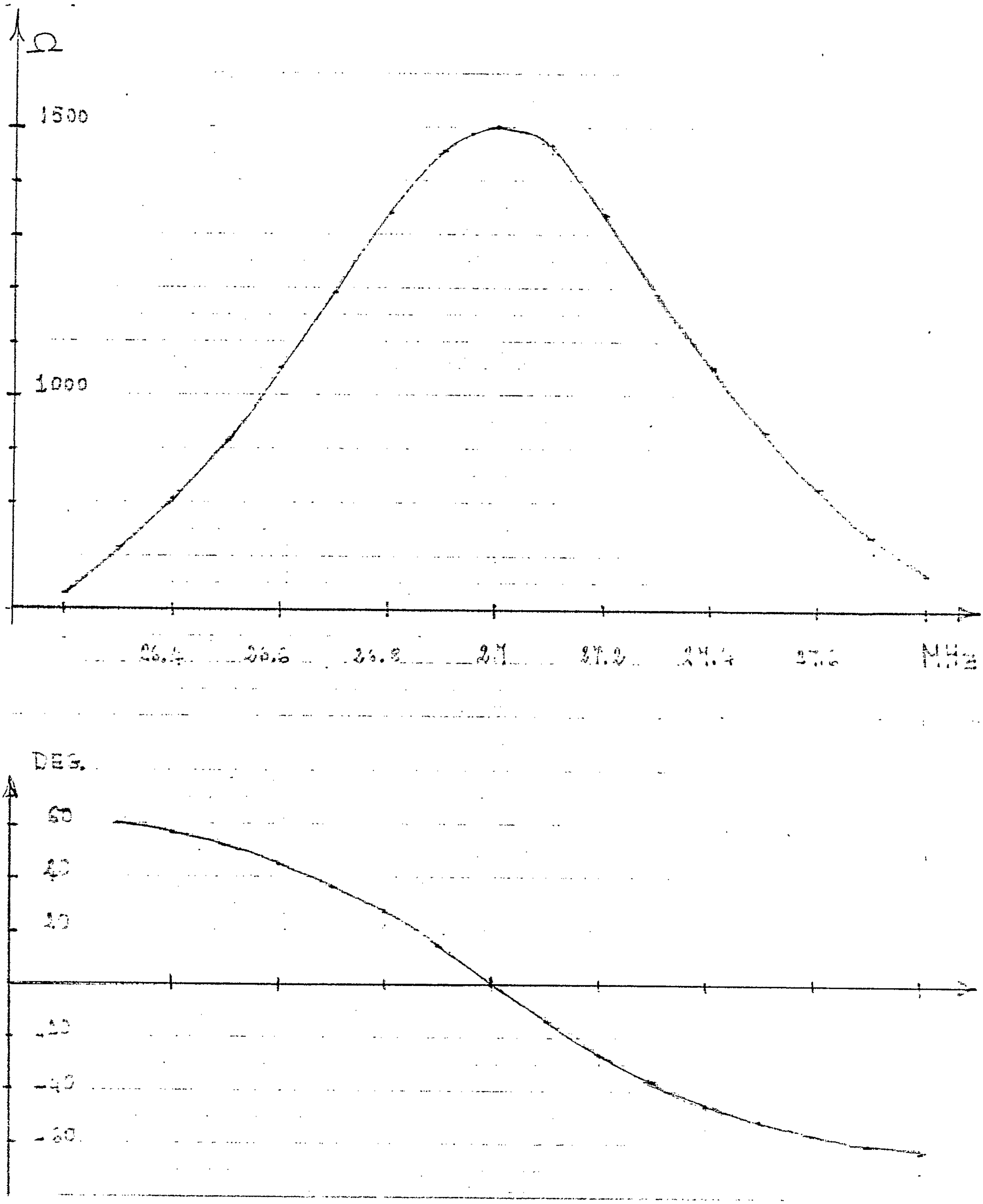

Fig. 3. Total impedance seen by the driven circuit.

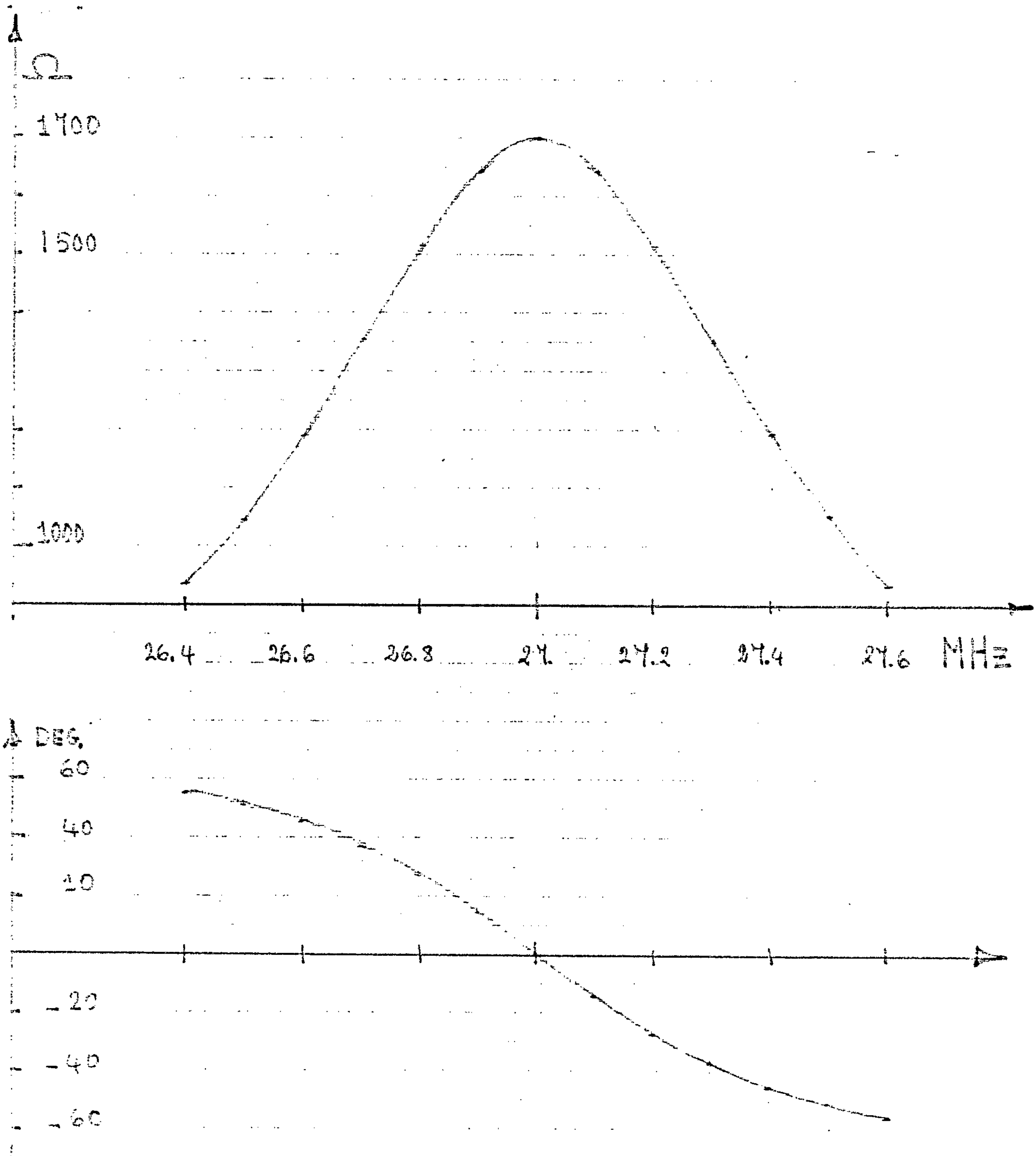

Fig. 4. Total impedance seen by the driver.